$\mathtt{P} \neq \mathtt{NP}$

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Let A be a countably infinite set of computable irrational numbers of polynomial time-complexity. Let Σ be a finite alphabet that allow us to represent algorithms (convergent sequences) for elements in A, including numbers 0-9 and decimal point. Let L be a language over Σ that contain all elements in A, up to any finite number of decimals.

Define a relation $R \subset \Sigma^* \times \Sigma^*$ by xRy if and only if $x \in A$ is an irrational number and $y = y(n)$ is an algorithm that calculates x up to n decimals, for any n.

Then $L \in \mathtt{NP}$, because there exists a positive integer k that satisfies:

1. For all $x \in \Sigma^*$, $x \in L \Leftrightarrow y \in \Sigma^*$ and $(x, y) \in R$ and $|y| \in O(|x|^k)$. The former follows immediately from how R was defined. Since $|y|$ is independent of x, the latter follows.
2. The language $L_R = \{x\#y : (x, y) \in R\}$ over $\Sigma \cup \{\#\}$ is decidable by a deterministic Turing Machine in polynomial time. Since we verify by running the algorithm y and checking that is equal to x, the running time is linear over $|x|$.

$L \notin \mathtt{P}$, because there exists no algorithm that can compute every computable irrational number. Such an algorithm would be infinitely long.