On Economic Coordination

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August 30, 2024

Abstract

In this paper, we investigate the special role played by currency, especially how prices are determined in an economy, and how prices relate to individual preferences.

It is near impossible to find a uniform template suitable to describe every good or service. However, we can exploit the fact that there is a game-theoretic equilibrium that prevents either counterparty to stray too much from a reasonable description of their agreement. We call such descriptions **economic statements**, and sets of these **economic units**.

Economic units can be constructed from other economic units. This also applies recursively, for a finite number of steps. The final "product" E is related somehow to these. This does not necessarily mean that E is the union of its inputs, but we at least assume that its intersection is non-empty.

Definition 1. A topological space X where the open sets are economic units is a **economic space**.

Such an economic space is an "mental representation" (an understanding) of how the world works. Our understanding changes over time as we interact with each other. We will index economic spaces by a time variable t where needed.

We denote the economic space associated to individual i at time t as X_t^i.

Axiom 2. Every individual has a unique economic space.

This amounts to saying that every individual has a unique experience and understanding. The degree to which two separate economic spaces differ is left unspecified, along with how useful that experience is to others.

Axiom 3. Every individual makes the best decision given their current understanding.

We will assume as given that every individual, upon deciding between different options, will always choose the option that allows her to retain the greatest amount of currency. It is understood that the set of feasable options can change over time.

Definition 4. Let $X_t^1, X_t^2, \ldots, X_t^n$ be economic spaces. An **economic outer measure** is an outer mesaure c_t defined on $X = \cap_i X_t^i$ taking values in the non-negative reals. A subset $E \subseteq X$ that satisfies

$$c_t(A) = c_t(A \cap E) + c_t(A \setminus E)$$

for every $A \subseteq X$, is said to be c_t**-measurable**.

Note that economic (outer) measures correspond to preferences, not to prices. To speak about the c_t-measurable sets on an intersection of two or more economic spaces is to speak about how the relative preferences of these economic spaces mutually agree.

Definition 5. Let $J \subseteq I$ be a set of individuals with economic spaces (X_t^i), c_t an outer measure defined on its intersection. The restriction of c_t to the c_t-measurable subsets is called an **economic measure**, denoted c_t^J.

Definition 6. A **quiver** G is an ordered 4-tuple $G = (V, A, s, t)$, where V is a set of vertices, A a set of edges, $s : A \to V$ assigning each edge its source node, $t : A \to V$ assigning each edge its target node.

Definition 7. Let c_t be an economic measure on X, the c_t-measurable economic units of a finite intersection of economic spaces. An **economic graph** is a pair $G = (G', X)$, where G' is a quiver where the vertices correspond to individuals, the edges correspond to c_t-measurable economic units in X.

Note that, given an edge $E \in G$ in an economic graph G, the source node $s(E)$ refers to the individual that recieves the profit of E, i.e. the seller, or in the case of work, the employee.

By *work* we mean *allocated time*. Note that the below definition also includes raw materials as this is included in work of a given economic unit. As long as both seller and buyer *agree* on what has been done, there is no need to e.g. distinguish between factors of production in our model.

Definition 8. Let G be an economic graph. A subgraph $G' \subseteq G$ is also an economic graph, with economic measure restricted to the edges of G'. G' is called an **economic subgraph**.

Definition 9. A **currency** is an economic outer measure c_t defined on an economic graph G. If c_t coincides with the economic measure of every economic subgraph of G, then we say that c_t is **sound**. Otherwise, it is said to be **unsound**.

Given an economic unit $E \in G$ in an economic graph G, there exists a subgraph of $G^E \subseteq G$ that is the *supply chain* of E. That is, its the necessary economic units (allocated time by individuals plus materials, both of which are represented by economic units) needed to produce E. Any supply chain, such as G^E, is topologically ordered, and as such, is a directed acyclic graph.

Definition 10. Let G be an economic graph and $E \in G$ an economic unit. The **supply chain** of E, denoted $S(E)$ is a directed acyclic graph that describes the dependencies necessary to produce E. $S(E)$ is a subgraph of G.

Definition 11. Let G_1, G_2 be two economic graphs. If every economic unit $E_1 \in G_1$ is measurable in G_2, and every economic unit $E_2 \in G_2$ is measurable in G_1, then we say that G_1, G_2 are **compatible**. If the intersection of their edge sets is empty, then G_1, G_2 are trivially compatible.

Every economic subgraph $G' \subseteq G$ is compatible with $G \setminus G'$. This follows immediately from the fact that the economic units in G are c^G-measurable by assumption.

Definition 12. Let G be an economic graph. If every individual i can enter any economic subgraph $G' \subseteq G$ in finite time t, then we say that G is **non-discriminatory**.

The property of an economic graph to be (non-)discriminatory applies recursively to all economic subgraphs.

Lemma 13. *Let $S(E)$ be a supply chain of E, G an economic graph, with currency c_t. If $S(E)$ and G are incompatible and non-discriminatory, then they will be compatible in finite time.*

Proof. There a couple of cases to untangle here. Note that selling or buying from another economic subgraph relies on it being non-discriminatory.

Case 1: There exists $A \in G$ such that

$$c_t^G(A) < c_t^G(A \setminus E) + c_t^G(A \cap E)$$

If $S(A) \cap G = \emptyset$, then there is a buyer i in G with a preference for E over A. Therefore, since G is non-discriminatory, $S(E)$ can sell to $i \in G$ at a higher price in finite time. If $S(A) \cap G \neq \emptyset$, then $S(A)$ has a buyer, not necessarily in G, with higher preference for E than A. In both cases: Either $S(E)$ chooses to sell at the same or a higher price, or $S(A)$ sells at a lower price.

Case 2: There exists $A \in G$ such that

$$c_t^{S(E)}(E) < c_t^{S(E)}(E \setminus A) + c_t^{S(E)}(E \cap A)$$

If $S(A) \cap G = \emptyset$, then there is a buyer in G that finds $A \in G$ more preferable than E. If $S(A) \cap G \neq \emptyset$, then $S(A)$ has a buyer, not necessarily in G, that prefers A over E. In both cases: Either $S(A)$ chooses to sell at the same or a higher price, or $S(E)$ sells at a lower price.

□

We call the process in (13) a *reconciliation process*. It is the idea that preferences are updated in an economy as new information emerge. We think of information as being "priced in" and propagated via prices by all the participants in an economy.

Corollary 14. *Let $S(E)$ be a supply chain of E, G an economic graph, with currency c_t. If $S(E)$ and G are incompatible and discriminatory, then c_t is unsound.*

Proof. Since $S(E)$ and G are incompatible and cannot be compatible in finite time. As mentioned in (13), the proof relies on the non-discriminatory property. Either $S(E)$ and G cannot choose to be compatible anyway, since it would contradict (3). So $S(E)$ and G are two economic subgraphs whose economic measures do not coincide with c_t. Therefore, c_t is unsound.

□

We now assume that there is an economic graph G, refered to as *an economy*, to which every economic graph is an economic subgraph of.

Corollary 15. *Let G be an economy with currency c_t. If G is non-discriminatory, then c_t is sound.*

Proof. This follows from (13) and (14).

□

An individual (or a company) is always better off focusing on high profit work. If we can specify an actionable description of the low profit work, we can delgate it. Or, perhaps it could even be carried out by a computer.

Consider an economic unit E that can be split into two parts E_1, E_2.

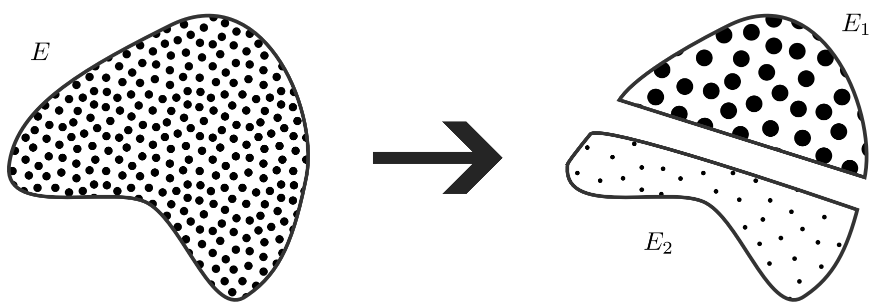

Figure 1: The decomposition of an economic unit

Let $c_t(A)$ be the price of A per unit of time. If $c_t(E_1) > c_t(E_2)$, then you are better off spending more time on E_1 than on both E_1, E_2. Because in this case, $c_t(E_1) < c_t(E) < c_t(E_2)$ *per unit of time.*

An economy therefore naturally seeks to price commensurate effort equivalently. We can still obtain a high profit by grouping low-effort work together. This phenomenon is typically called *division of labour* and has been observed by economists throughout history.